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Integro-Differential Equations: Asymptotic Properties and Controllability

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Abstract

In this work, we present a contribution to study of asymptotically almost automorphic solutions for integro-differential equations. We establish conditions that guarantee the existence of mild-weak solutions. One of the main research directions focuses on several existence results, exact controllability, trajectory controllability, and dependence on initial conditions for certain classes of integro-differential equations. Using the resolvent operator in Grimmer's sense, we analyze these equations in various contexts (without impulses, with instantaneous and non-instantaneous impulses, with local and non-local conditions, with infinite delay, and on unbounded intervals). Moreover, we generalize these results to stochastic integro-differential equations in Hilbert spaces. The existence results are obtained through the use of fixed-point theorems, combined with Kuratowski's measure of non-compactness as well as De Blasi's measure of non-compactness, exploiting weak topology and Pettis integral in Banach spaces, whether reflexive or not, as well as in Fréchet spaces.

Key words and phrases: Mild solution, Asymptotically Almost Automorphic mild Solution, approximate controllability, Trajectory Controllability, Pettis integrable, integrodifferential equation, impulsive, nonlocal conditions, infinite delay, weak topology, fixed point, measures of noncompactness, De Blasi measure of weak noncompactness, stochastic processes, resolvent operator, Fréchet space, Banach space, Hilbert space.

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