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On the existence and multiplicity of solutions for second-order impulsive differential equations with mixed impulses

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Abstract :

This work explores a class of nonlinear second-order differential equations influenced by both instantaneous and non-instantaneous impulsive effects. Impulsive differential equations are a powerful tool for modeling dynamic systems that experience abrupt changes. These changes can represent real-world phenomena like sudden population shifts, power outages, economic jolts from policy interventions.

Within this framework, two main types of impulses are studied:

- Instantaneous impulses: describe sudden, brief changes in a system's state. The change occurs and ceases almost immediately.
- Non-instantaneous impulses: involve changes that occur over a period of time before the system returns to its initial state.

In our research, we focus on a specific category of nonlinear second-order differential equations. These equations exhibit two distinct types of impulses: instantaneous and non-instantaneous. Our investigation centers around the following problem

$$\begin{cases}
-x''(t) + \Lambda_i(t)x(t) = \sigma_i(t, x(t)), & t \in (e_i, s_{i+1}], i = \overline{0, m}, \\
x'(s_i^+) - x'(s_i^-) = \Theta_i(x(s_i)), & i = \overline{1, m}, \\
x'(t) = x'(s_i^+), & t \in (s_i, e_i], i = \overline{1, m}, \\
x'(e_i^+) = x'(e_i^-), & i = \overline{1, m}, \\
x(0) = x(T) = 0.
\end{cases}$$
(1)

Where $0 = e_0 < s_1 < e_1 < s_2 < e_2 < \ldots < s_m < e_m < s_{m+1} = T$, $\Lambda_i \in L^{\infty}(e_i, s_{i+1}]$, σ_i are Carathéodory functions defined on $(e_i, s_{i+1}] \times \mathbb{R}$, $x'(s_i^{\pm}) = \lim_{t \to s_i^{\pm}} x'(t)$, $\Theta_i \in \mathcal{C}(\mathbb{R})$. In these equations, instantaneous impulses occur at points s_i , whereas non-instantaneous impulses persist over intervals $(s_i, e_i]$ while maintaining a constant derivative. We employ variational methods and critical point theory to investigate the existence and multiplicity of weak solutions.

Keywords:

Keywords : Impulsive differential equations, instantaneous impulses, non-instantaneous impulses, variational methods, energy functional, critical point theory.

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