# Séminaire de Mathématiques et Informatique

## Université Djilali Liabès - Sidi Bel Abbès - le 15 février 2025

### Fractional Order Differential Equations : Analysis and Stability

Nawal BETTAYEB

LDM - UDL

**Abstract**

In this work, we present a contribution to study of various classes of initial, implicit and implicit neutral fractional differential equations, inclusions and a coupled systems as well as boundary value problems of fractional differential equations involving the Caputo tempered fractional derivative and the tempered $\psi$-Caputo fractional derivative with local and nonlocal conditions, with and without delay ( there cases are studied finite, infinite, and state-dependent delays) and some problems are studied with nonlinear integral conditions on bounded and an unbounded positive interval. The arguments used in this work are based on fixed point theorems of Krasnoselskii, Schauder, Martelli as well as the approach involving upper and lower solutions and the diagonalization process. while some results are based on the concept of measure of weak noncompactness combined with Mönch's type fixed point theorem as well as on the concept of the degree of nondensifiability (DND) combined with Darbo-type fixed point theorem in Banach space. We also discuss the global convergence and uniqueness properties of the successive approximations method, Ulam stability results and oscillatory and nonoscillatory results for some problems. To illustrate our obtained results, we provide some examples every chapter.

**Key words and phrases:** The Caputo tempered fractional derivative, the tempered $\psi$-Caputo fractional derivative, fractional differential equations, fractional differential inclusions, implicit neutral problem, coupled system, boundary value problem, existence, uniqueness, Ulam stability, solution, bounded solution, weak solution, upper solution, lower solution, global convergence, successive aproximations, nonlocal conditions, finite delay, infinite delay, sate dependent delay, oscillation, nonoscillation, diagonalization process, Pettis integral, measure of weak noncompactness, degree of nondensifiability, fixed point, Banach space.
**AMS (MOS) Subject Classifications**: 34A08, 34B15, 34C10, 34C15, 34K37, 65L05.

## References

[1] S. Abbas, B. Ahmad, M. Benchohra and A. Salim, *Fractional Difference, Differential Equations and Inclusions: Analysis and Stability*, Morgan Kaufmann, Cambridge, 2024.

[2] S. Abbas, A. Arara, M. Benchohra, Global convergence of successive approximations for abstract semilinear differential equations. *PanAmer. Math. J.* **29** (2019), no. 1, 17-31.

[3] R. Almeida, M. L. Morgado, Analysis and numerical approximation of tempered fractional calculus of variations problems, *J. Comput. Appl. Math.* **361** (2019), 1-12.

[4] M. Benchohra, S. Hamani and J. Henderson, Functional differential inclusions with integral boundary conditions, *Eletron. J. Qual. The. Differ. Equa.* **2007** (15) (2007), 13 pp.

[5] M. Benchohra, E. Karapınar, J. E. Lazreg and A. Salim, *Advanced Topics in Fractional Differential Equations: A Fixed Point Approach*, Springer, Cham, 2023.

[6] M. Benchohra, F-Z. Mostefai, Weak solutions for nonlinear fractional differential equations on reflexive Banach spaces, *Electronic Journal of Qualitative Theory of Differential Equations* **54** (2010), 1-10.

[7] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, Boundary Value Problem for Caputo Tempered Fractional Differential Equations Under the Weak Topology. (**Submitted**)

[8] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, Existence and Oscillatory Results for Caputo Tempered Fractional Differential Equations and Inclusions. *Mathematica Applicanda*, 52(1) 2024, 149–171

[9] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, Existence and Oscillatory Results of Differential Coupled Systems with Caputo Tempered Fractional Derivative. (**Submitted**).

[10] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, Existence Results of Differential Coupled Systems with Implicit Tempered $\psi$-Caputo Fractional Derivative via Densifiability Techniques . (**Submitted**).

[11] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, Existence, Uniqueness and Successive Approximations for Implicit Caputo Tempered Fractional Differential Equations. (**Submitted**).

[12] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, On Implicit Neutral Caputo Tempered Fractional Differential Equations with Delay. *Applied Mathematics E-Notes.* 24 (2024), 424-442.

[13] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, On implicit neutral tempered $\psi$-Caputo fractional differential equations with delay via densifiability techniques. *Adv. Theory Nonl. Anal. Appl.* **7**(5) (2023), 44-65.

[14] N. Bettayeb, A. Salim, J. E. Lazreg and M. Benchohra, On Tempered $\psi$-Caputo Fractional Differential Boundary value Problem via Densifiability. *Nonlinear Dynamics and Systems Theory.* (2024) (**Accepted**).

[15] F. S. De Blasi. On the property of the unit sphere in a Banach space. *Bull. Math. Soc. Sci. Math. RS Roumanie* **21** (1977), 259-262.

[16] G. García, G. Mora, A fixed point result in Banach algebras based on the degree of nondensifiability and applications to quadratic integral equations. *J. Math. Anal. Appl.* **472** (2019), 1220-1235.

[17] D. H. Hyers, On the stability of the linear functional equation. *Proc. Nat. Acad. Sci. U.S.A.* **27**, (1941), 222-224.

[18] J. E. Lazreg, M. Benchohra and A. Salim, Existence and Ulam stability of $k$-generalized $\psi$-Hilfer fractional problem. *J. Innov. Appl. Math. Comput. Sci.* **2** (2022), 01-13.

[19] C . Li, W. Deng and L. Zhao, Well-posedness and numerical algorithm for the tempered fractional differential equations, *Discr. Contin. Dyn. Syst. Ser. B.* **24** (2019), 1989-2015.

[20] B. J. Pettis. On integration in vector spaces. *Trans. Amer. Math. Soc.* **44** (1938), 277-304.

[21] A. Salim, B. Ahmad, M. Benchohra and J. E. Lazreg, Boundary value problem for hybrid generalized Hilfer fractional differential equations, *Differ. Equ. Appl.* **14** (2022), 379-391.

[22] A. Salim, M. Benchohra, E. Karapınar and J. E. Lazreg, Existence and Ulam stability for impulsive generalized Hilfer-type fractional differential equations. *Adv. Differ. Equ.* **2020** (2020), 21 pp.

[23] Y. Zhou, *Basic Theory of Fractional Differential Equations*, World Scientific, Singapore, 2014