Séminaire de Mathématiques et Informatique

Université Djilali Liabès - Sidi Bel Abbès - le 19 mars 2022

A multivalued version of Krasnosel'skii type compression fixed point theorem for set contractions and star convex sets

Amirouche LAADJEL[a] & Abdelghani OUAHAB[b]

[a] Université Larbi Tebessi, Tebessa 12002 [b] Université Djilali Liabès, Sidi Bel Abbès, 22000

Abstract

We prove some fixed point theorems on convex and star convexe sets. More precisely, we present two multivalued versions of Krasnosel'skii compression fixed point theorem and apply their results to implicit differential equation.

Krasnosel'skii compression-expansion fixed point theorem is a powerful tool to prove the existence of positive solutions to several classes of boundary value problems and also to obtain multiple solutions see [10, 11, 12, 13, 14, 15], where for applications, it is more convenient to use Krasnosel'skii theorem, because it offers directly the conditions of compression or expansion that have to be verified.

The direct approach owed to Krasnosel'skii was followed by Potter [4], who extended the compression result from compact mappings to set contractions as follow, if $(X, \|\cdot\|)$ is a Banach space and C a cone in X, given two real numbers r, R with $0 < r < R$, denote

$$F_{r,R} = \{x \in C : r \leq \|x\| \leq R\}.$$
$$B_r = \{x \in C : \|x\| \leq r\}.$$
$$S_r = \{x \in C : \|x\| = r\}.$$

Recalling Potter's compression result for balls that is, if $T : F_{r,R} \to C$ is a k-set contraction with $0 \leq k < 1$ and a compression of the cone C, i.e.,

$$x - T(x) \notin C, \text{ for all } x \in S_r,$$
$$T(x) - (1+\varepsilon)x \notin C, \text{ for all } \varepsilon > 0 \text{ and } x \in S_R,$$

Then T has at least one fixed point in $F_{r,R}$.

keywords : Multivalued map, fixed point, compression, star convex sets, set contraction, implicit differential equation.

References

[1] PRECUP, R., Methods in Nonlinear Integral Equations, *Kluwer Academic Publishers Amsterdam,* (2002).

[2] SMAÏL DJEBALI AND LECH GÓRNIEWICZ AND ABDELGHANI OUAHAB, Solution Sets for Differential Equations and Inclusions, *De Gruyter* (2012).

[3] GRAEF, JOHN R.; HENDERSON, JOHNNY; OUAHAB, ABDELGHANI, Topological methods for differential equations and inclusions, *CRC Press* (2019).

[4] A. J. B. POTTER, A fixed point theorem for positive k-set contractions, *Proceedings of the Edinburgh Mathematical Society,* **19(1)** (1974), 93–102.

[5] M.A. KRASNOSELSKY, Positive Solutions of Operator Equations, *P. Noordhoff* (1964).

[6] LOIS-PRADOS, C., AND RODRÍGUEZ-LÓPEZ, R., A generalization of Krasnosel'skii compression fixed point theorem by using star-convex sets, *Proc. Royal Soc. Edinburgh,* **150(1)** (2020), 277–303.

[7] LOIS-PRADOS, C., PRECUP, R. AND RODRÍGUEZ-LÓPEZ, R., Krasnosel'skii type compression-expansion fixed point theorem for set contractions and star convex sets, *J. Fixed Point Theory Appl.* **22,63** (2020).

[8] J. BANAŚ AND K. GOEBEL, Measures of Noncompactness in Banach Spaces, Lecture Notes in Pure and Applied Mathematics, *Marcel Dekker, New York,* (1980).

[9] R. Precup; Methods in Nonlinear Integral Equations, *Kluwer, Dordrecht*, (2002).

[10] ERBE, L.H., WANG, H., On the existence of positive solutions of ordinary differential equations, *Proc. Am. Math. Soc,* **120** (1994), 743–748.

[11] GUO, D., LAKSHMIKANTHAM, V., LIU, X., Nonlinear Integral Equations in Abstract Spaces, *Springer, New York* (2013).

[12] LIAN, W.C., WONG, F.H., YEH, C.C., On the existence of positive solutions of nonlinear second order differential equations, *Proc. Am. Math. Soc,* **124** (1996), 1117–1126.

[13] O'Regan, D., Precup, R., Compression-expansion fixed point theorem in two norms and applications, *PJ. Math. Anal. Appl,* **309** (2005), 383–391.

[14] Torres, P.J., Existence of one-signed periodic solutions of second-order differential equations via a Krasnosel'skii fixed point theorem, *J. Differ. Equ,* **190** (2003), 643–662.

[15] Zima, M., Fixed point theorem of Legget–Williams type and its application, *J. Math. Anal. Appl,* **299** (2004), 254–260.

Amirouche Laadjel

Department of Mathematics and Computer Science, Larbi Tebessi University, 12002 Tebessa

Laboratory of Fixed Point Theory and Application, ENS Kouba, PoBox 92, 16006 Algiers

amir_ laadjel@gmail.com

Abdelghani Ouahab

Laboratory of Mathematics, Sidi-Bel-Abbès University, PoBox 89, 22000 Sidi-Bel-Abbès

agh_ ouahab@yahoo.fr