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# On k-balancing and k-cobalancing numbers

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**Abstract :**

$B \in \mathbb{N}^*$ is called a balancing number (respectively cobalancing number) if the Diophantine equation

$$1 + 2 + \ldots + (B - 1) = (B + 1) + (B + 2) + \ldots + (B + s) \tag{1}$$

$\Big($respectively

$$1 + 2 + \ldots + (b - 1) + b = (b + 1) + (b + 2) + \ldots + (b + s) \tag{2}$$

$\Big)$

holds for some positive integer $s$ which is called *balancer* (respectively cobalancer) corresponding to $B$ (respectively $b$) [1].

One finds the successive solutions of (1) (resp. (2)), iff $\sqrt{8\,B^2 + 1}$ (resp. $\sqrt{8\,b^2 + 8\,b + 1}$) is a perfect square. Balancing (resp. cobalancing) numbers verify the recursive equation

$B_{n+1} = 6B_n - B_{n-1}$ (resp. $b_{n+1} = 6b_n - b_{n-1} + 2$.

More general balancing numbers can be extracted also from solutions of the Diophantine equation

$$1^h + 2^h + \ldots + (B - 1)^h = (B + 1)^l + (B + 2)^l + \ldots + (B + s)^l.$$

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We define $k$-balancing numbers by the sequence $(B_{k,n})_n$ which verifies recursively

$B_{k,n+1} = 6kB_{k,n} - B_{k,n-1}$ with the initials $B_{k,0} = 0$ and $B_{k,1} = 1$.

In this presentation, we give some properties of k-balancing and k-cobalancing numbers.

**Keywords : Balancing numbers, Cobalancing numbers, Diophantine Equations, k-balancing numbers, k-cobalancing numbers**

**Mathematics Subject Classification : 11Bxx, 11Dxx, 11D59, 11Yxx**

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