

# SÉMINAIRE DE MATHÉMATIQUES ET INFORMATIQUE

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## A Fixed Point Theorem for Multi-Valued Mappings in Two Generalized Metric Spaces

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### Abstract

We introduce several local and global fixed-point theorems for multivalued mappings with a generalized Perov contraction. The results are based on Perov type fixed point theorems in spaces endowed with two generalized metric spaces. Then, we present a homotopy result for single-valued mappings with a generalized Perov contraction on a generalized metric space. Finally, we provide an application to a boundary value problem in a Banach space.

**Mathematics Subject Classification :** 54H25, 47H10, 47H04.

**keywords :** Multi-valued map; generalized metrics; fixed point, matrix convergent to zero.

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