A multiplicity Result for a Q-curvature type Problem on Closed Riemannian Manifold

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Abstract

Given a compact Riemannian manifold (M, g) of dimension $n \geq 3$ without boundary, using variational methods, we study the existence of solutions for the elliptic equation

$$P_g^k u = f|u|^{N-2}u + \lambda h|u|^{q-2}u, \tag{1}$$

where P_g^k is the GJMS operator of order $2k < n$, $h, f \in C^\infty(M)$, $1 < q < 2$, $\lambda > 0$ and N is the critical Sobolev exponent for the space $H_k^2(M)$. We apply Ljusternik-Schnirelmann theory on C^1-manifolds to prove that under some conditions, the equation (1) admits infinitely many solutions.

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References

[1] A. Ambrosetti, A. Malchiodi, *Nonlinear Analysis and Semilinear Elliptic Problems*, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2007, 157–176.

[2] T. Aubin, *Some Nonlinear Problems in Riemannian Geometry*, Springer Berlin, Heidelberg, 1998, 145–250.

[3] T.P. Branson, *Origins, applications and generalisations of the Q-curvature*, Acta Appl. Math. 102, no. 2-3 (2008), 131–146.

[4] M. Benalili, K. Tahri, *Multiple solutions to singular fourth order elliptic equations on compact manifolds*, Complex Variables and Elliptic equations. 60 (2015), 1621–1646.

[5] M. Benalili, K. Tahri, *Nonlinear elliptic fourth order equations existence and multiplicity results*, Nonlinear Difer. Equ. Appl. 18 (2011), 539–556.

[6] P. Esposito, F. Robert, *Mountain pass critical points for Paneitz-Branson operators*, Calc. Var. 15 (2002), 493–517.

[7] C.R. Graham, R. Jenne, L.J. Mason, G.A.J. Sparling, *Conformally invariant powers of the Laplacian*, I. Existence. J. London Math. Soc. 46 (1992), 557–565.

[8] A. Szulkin, *Ljusternik-Schnirelmann theory on C^1-manifolds*, Annales de l'I.H.P. Analyse non linÃ©aire 5.2 (1988), 119–139.

[9] S. Mazumdar, *GJMS-type operators on a compact Riemannian manifold: best constants and Coron-type solutions*, J. Differential equations 261, no. 9 (2016), 4997–5034.

[10] M.E. Ounane, K. Tahri, *Multiplicity of solutions for an elliptic problem involving GJMS operator*, Filomat 38:5 (2024), 1547–1569.

[11] F. Robert, *Admissible Q-Curvatures under isometries for the conformal GJMS Operator*, arXiv:1003.0612 (2010).

[12] C.A. Swanson, *The best Sobolev constant*, Applicable Analysis 47(1-4) (1992), 1227–239.