Solutionnaire de l'examen EEC (24/25)

Exo1 : $\vec{V} = x^2\vec{i} - 2xy\vec{j}$ (6pts)

1) Vérification de la conservation de la masse :

l'écoulement est conservatif si $\operatorname{div}\vec{V} = 0$ (1) $\vec{V}\begin{pmatrix} x^2 \\ -2xy \end{pmatrix}$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(-2xy) = 0$$

$$\Rightarrow 2x - 2x = 0 \Rightarrow \boxed{0 = 0}$$ (2) ⇒ la conservation de la masse est vérifiée.

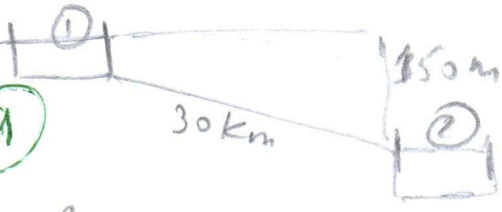
2) L'équation des lignes de courant :

On donne : $u = \frac{\partial \psi}{\partial y}$ et $v = -\frac{\partial \psi}{\partial x}$. (1)

$$\Rightarrow \begin{cases} x^2 = \frac{\partial \psi}{\partial y} \\ -2xy = -\frac{\partial \psi}{\partial x} \end{cases} \Rightarrow \begin{cases} \partial\psi = x^2\partial y \\ \partial\psi = 2xy\,\partial x \end{cases} \Rightarrow \begin{cases} \psi = x^2 y + c_1 \\ \psi = \frac{2x^2}{2} y + c_2 \end{cases}$$

$$\Rightarrow \boxed{\psi = x^2 y + c}$$ (2)

Exo2 (6pts) : $L = 30\,km$, $\Delta Z = 150\,m$, $Q = 5000\,m^3/J$; $\varepsilon = 0{,}3\,mm$. $D = ?$



1) l'éq de Bernoulli entre les 2 réservoirs donne :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \Delta h_{1-2}$$ (1)

$$\Delta h_{1-2} = \Delta Z \Rightarrow \frac{\lambda L}{D}\frac{V^2}{2g} = \Delta Z \Rightarrow \frac{8\lambda L}{\pi^2 g D^5} Q^2 = \Delta Z.$$

$$\Rightarrow \frac{8\lambda \cdot 30000}{\pi^2 \cdot 9{,}81 \cdot D^5}\left(\frac{5000}{86400}\right)^2 = 150 \Rightarrow \boxed{D = 0{,}56\,\lambda^{0{,}2}}$$ (2)

$Re = \frac{4\rho Q}{\pi \mu D}$

λ	D	Re	λ (Colebrook)
0,01			
0,0212	0,259	284318	0,0212

(1)

2) Après itération $\boxed{D = 259\,mm}$ (2)

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EXO2 : L = 2,5 Km, D = 450 mm, Q = 550 l/s, (8 pt)

1) Calcul de ΔP. t = 8 s.

le bilan des forces appliquées à la conduite s'écrit $\Sigma F = m \frac{dv}{dt}$

Considérant la conduite horizontale : $\Rightarrow P_1 A_1 - P_2 A_2 = \rho \bar{V} \frac{dv}{dt}$

$A_1 = A_2$.

$\Rightarrow (P_1 - P_2) A = \rho A L \left(\frac{0 - v}{t} \right)$ (1)

$\Delta P = \frac{\rho V L}{t}$ $Q = V.A = V.\frac{\pi D^2}{4} \Rightarrow V = \frac{4Q}{\pi D^2} = \frac{4.0{,}55}{\pi\, 0{,}45^2}$

$\Rightarrow$ $V = 3{,}46\ m/s$ ← (1)

$\Rightarrow \Delta P = \frac{10^3 . 3{,}46 . 2{,}5 . 10^3}{8} = 1080682\ pa$

Soit $\Delta P = 10{,}8\ bar$ (1)

2°) $\Delta P = \rho . C . v$ et $C = \sqrt{\frac{K}{\rho}} \Rightarrow C = \sqrt{\frac{2{,}2 . 10^9}{10^3}} = 1483\ m/s$ (1)

$\Rightarrow \Delta P = 10^3 . 1483 . 3{,}46 \Rightarrow \Delta P = 51{,}3\ bar$ (1)

3°) $\Delta P = \rho . C . v$ et $C = \sqrt{\frac{K'}{\rho}}$ $K' = \left(\frac{1}{K} + \frac{D}{E e} \right)^{-1} = \left(\frac{1}{2{,}2 . 10^9} + \frac{0{,}45}{3 . 210 . 10^9 . 5 . 10^{-3}} \right)^{-1}$

$\Rightarrow K' = 1{,}27 . 10^9$

$\Rightarrow C = \sqrt{\frac{1{,}27 . 10^9}{10^3}} = 1127\ m/s$ (1)

$\Delta P = 10 . 1127 . 3{,}46 \Rightarrow \Delta P = 38{,}98\ bar$ (1)

4°) le temps minimal pour une fermeture lente :

$t \geq \frac{2L}{C} = \frac{2 . 2500}{1127} = 4{,}44\ s$

(0) (1)