

1/3

CORRIGÉ TYPE DE
L'EXAMEN DU TRAITEMENT
DE SIGNAL

Exercice 1 (4 pt)

1) $y(t) = x(t - 0,5)$ (0,5) $y(t) = \frac{dZ(t)}{dt}$ (0,5)

2) retard de 0,5 $\Rightarrow Y(\omega) = e^{-0,5j\omega} X(\omega)$
 $Y(\omega) = e^{-0,5j\omega} \frac{\sin \omega/2}{\omega/2}$ (1)

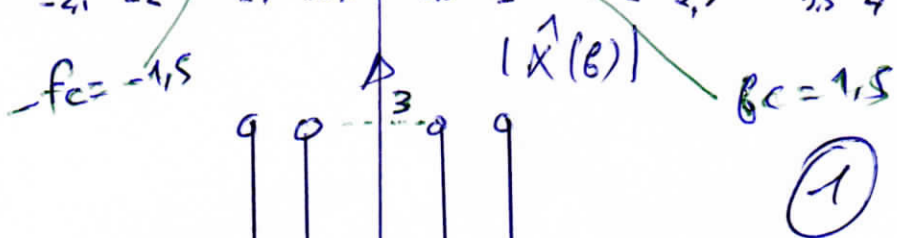
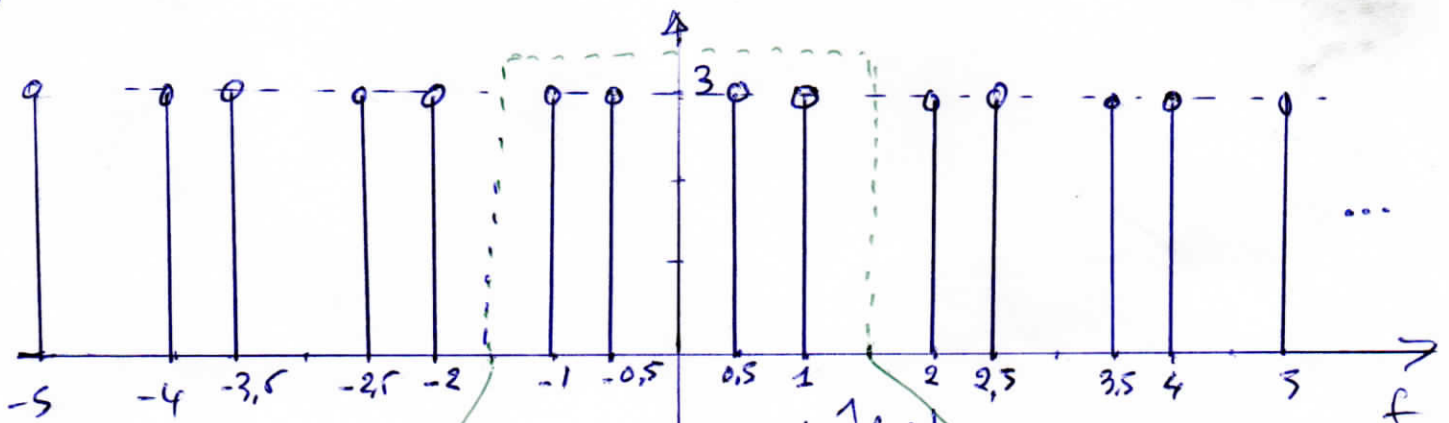
3) dérivée première $\Rightarrow Y(\omega) = j\omega Z(\omega)$
 $\Rightarrow Z(\omega) = \frac{Y(\omega)}{j\omega} = \frac{e^{-0,5j\omega} \sin \omega/2}{\omega^2/2}$ (1)

4) $E_z = \int_{-\infty}^{+\infty} Z^2 dt = \int_0^1 t^2 dt = \left. \frac{t^3}{3} \right|_0^1 = \frac{1}{3}$ (1)

Exercice 2 (4 pt)

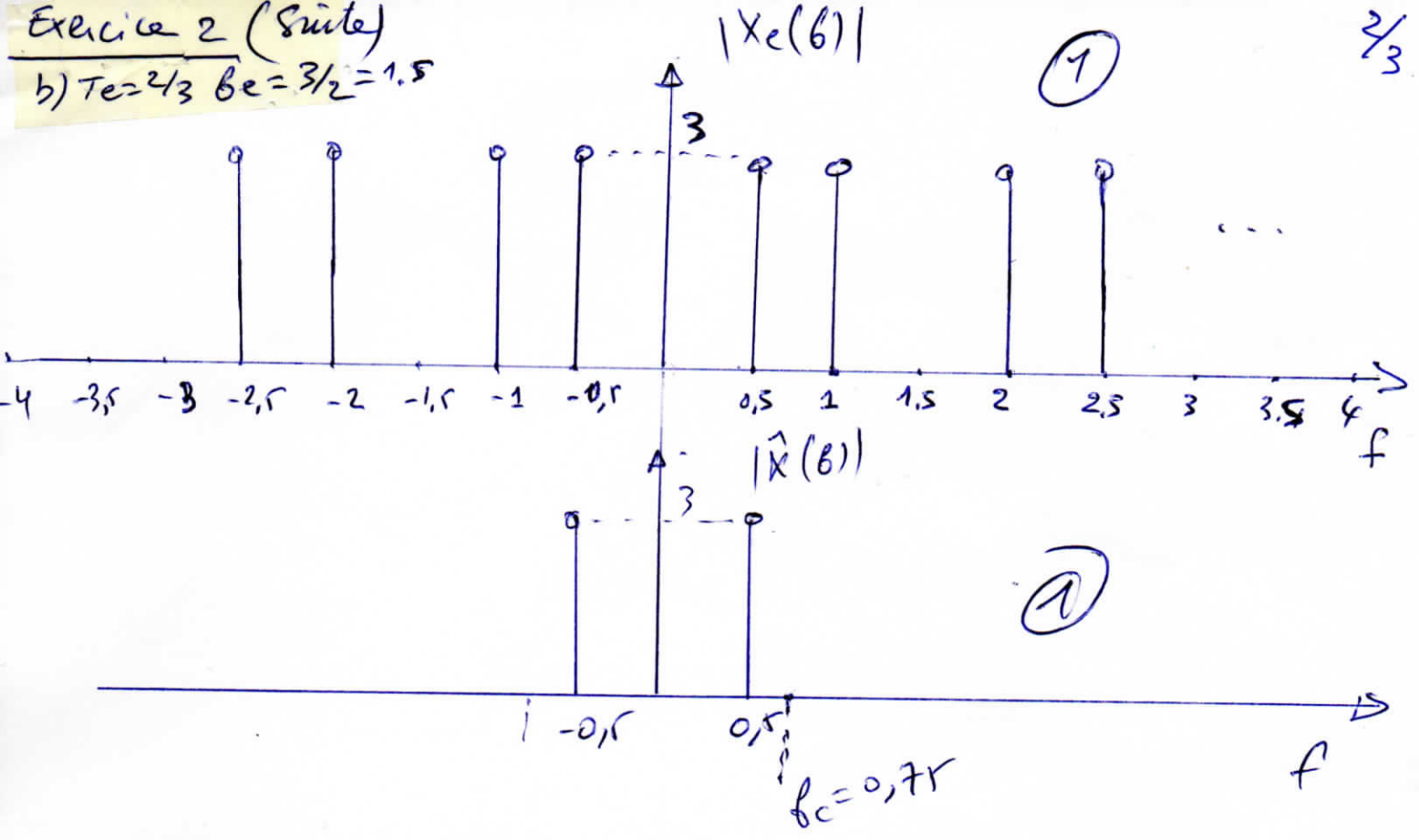
a) $T_e = 1/3$; $f_e = 3$

$|X_e(b)|$ (1)



Exercice 2 (suite)

b) $T_e = 2/3$ $B_e = 3/2 = 1.5$



Exercice 3 (5 pt)

1) $H(\omega) = \frac{1}{3+j\omega}$; $X(\omega) = \frac{1}{1+j\omega}$; $h(t) = u(t) e^{-3t}$

$$Y(\omega) = \frac{1}{(3+j\omega)(1+j\omega)} = \frac{-1/2}{3+j\omega} + \frac{1/2}{1+j\omega}$$
 (1)

$$\Rightarrow y(t) = \frac{u(t)}{2} (e^{-t} - e^{-3t})$$
 (0,5)

$$y(t) = \int_{-\infty}^t x(\tau) h(t-\tau) d\tau = \int_0^t e^{-2\tau} e^{-3(t-\tau)} e^{3\tau} d\tau \quad t \geq 0$$
 (0,5)

$$y(t) = e^{-3t} \int_0^t e^{2\tau} d\tau = \frac{1}{2} (e^{-t} - e^{-3t}) ; t \geq 0$$
 (0,5)

2) $x(t) = e^{2jt} = e^{j\omega_0 t}$; $\omega_0 = 2$

$$y(t) = |H(2)| e^{j(2t + \phi)}$$
 (0,5) $\phi = \arg(H(2))$

$$|H(2)| = \frac{1}{|3+2j|} = \frac{1}{\sqrt{13}}$$
 ; $\phi = -\arctan(\frac{2}{3}) = 0.6$ (0,5)

$$y(t) = \frac{1}{\sqrt{13}} e^{j(2t + 0.6)}$$
 (0,5)

Exercice 3 (suite)

3) il s'agit d'une réponse harmonique $\textcircled{1}$ 3/3

Exercice 4

$$H(z) = \frac{z}{z+0,5}; \quad h(n) = (-0,5)^n; \quad x(n) = 1; \quad X(z) = \frac{z}{z-1}$$

a) T. z: $Y(z) = H(z)X(z) = \frac{z^2}{(z+0,5)(z-1)} \quad \textcircled{0,5}$

$$Y(z) = \left(\frac{1}{3}\right) \frac{z}{z+0,5} + \left(\frac{2}{3}\right) \frac{z}{z-1} \Rightarrow y(n) = \frac{1}{3}(-0,5)^n + \frac{2}{3} \quad \textcircled{1}$$

b) par Conv. discr.

$$y(n) = \sum_{m=0}^n x(m)h(n-m) = \sum_{m=0}^n h(m)x(n-m) = \sum_{m=0}^n (-0,5)^m \quad \textcircled{0,5}$$

$$y(n) = \frac{1 - (-0,5)^{n+1}}{1+0,5} = \frac{1 + (0,5)(-0,5)^n}{1,5} = \frac{2}{3} + \frac{1}{3}(-0,5)^n \quad \textcircled{1}$$

b) c'est un filtre RII $\textcircled{0,5}$

$$|H(1)| = \frac{2}{3} \quad |H(-1)| = 2 \Rightarrow \text{passe-haut} \quad \textcircled{0,5}$$

c) $H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+0,5z^{-1}} \Rightarrow Y(z) = X(z) - 0,5z^{-1}Y(z)$
 $\Rightarrow y(n) = x(n) - 0,5y(n-1) \quad \textcircled{1}$

d) $H(z) = \frac{z}{z-p_1}$; $p_1 = -0,5$ $|p_1| = 0,5 < 1 \Rightarrow$ filtre stable $\textcircled{1}$

e) $X(\infty) = \lim_{z \rightarrow 1} \left(\frac{z-1}{z}\right)Y(z) = \lim_{z \rightarrow 1} \frac{z}{z+0,5} = \frac{2}{3} \quad \textcircled{1}$

ou encore $y(\infty) = \lim_{n \rightarrow \infty} y(n) = \frac{2}{3}$